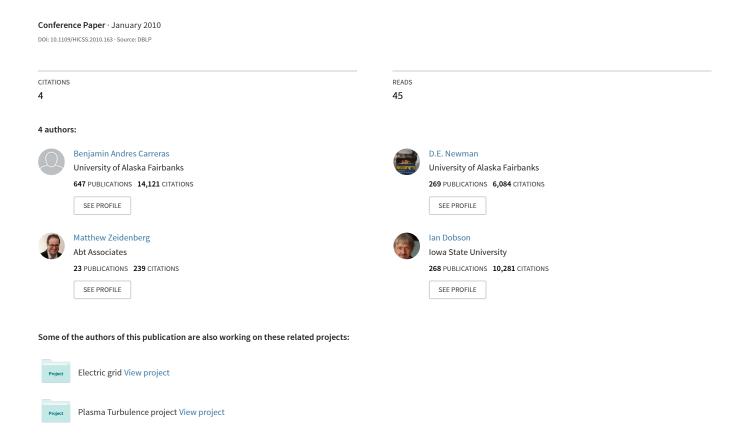
# Dynamics of an Economics Model for Generation Coupled to the OPA Power Transmission Model



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#### Abstract

In this paper we explore the interaction between a dynamic model of the power transmission system (OPA) and a simple economic model of power generation development. Despite the simplicity of this economic model, complex dynamics both in the economics (prices, market share etc) and in the transmission system characteristics (blackouts, reliability etc) are found. Depending on the values of the control parameters (the price enhancement factor, the critical margin and the Minimal Acceptable Rate of Return) the system can be in various states with vastly differing properties. These states are characterized by power law tails in the failure sizes in one limit and exponential tails with extremely high frequency of failures in the other limit. At least some of these control parameters can be thought of as regulatory based and could therefore be directly influenced by reliability considerations.

#### 1. Introduction

The OPA model [1, 2] was developed to study the failures of a power transmission system under the dynamics of an increasing power demand and the engineering responses to failure. In this model, the power demand is increased at a constant rate and was also modulated by random fluctuations. However, there was no dynamic mechanism regulating the increase in the generation of power. Rather, the generation power was automatically increased when the capacity margin was below a given critical level.

Using the OPA model we have been able to study and characterize the mechanisms behind the power tails in the distribution of the blackout size. These algebraic tails obtained in the numerical calculations are consistent with those observed in the study of the blackouts for real power systems [3, 4]. This model also permits us to separate the underlying causes for

cascading blackouts from the triggers that generate them. However there are clearly many important feedback mechanisms that are left out of OPA.

To address some of these mechanisms, and thereby improve the realism of the model, we have extended the model by including a minimal dynamical model for increasing the generation capacity as a response to the increasing demand. This extension has required us to include: 1) an electricity market that determines a daily electricity price and 2) a model for investment in new generation based on economical incentives.

In this paper, we describe this simple economic model for generation and the impact on the reliability of the system from the changes introduced in this model.

### 2. The electricity market

In the new version of the OPA model, we model a system with several independent utilities. Each utility is responsible for one or more generation nodes in the network. With this framework, we have set up a daily electricity market. The type of market that best fits our purpose is an electricity pool [5].

Let us consider a system with  $N_g$  generators. They produce electricity at different costs. Each makes a daily offer to sale, which we assume equal to their cost of producing electricity. One could device a more sophisticated market process, but for our purpose it is sufficient. Given these offers and the amount of electricity the generators can produce, the supply function can be constructed. Since the elasticity of demand is very low, we take it to be a constant value. In this manner, the daily clearing price is determined. In Fig. 1, we show an example of the supply function used in the OPA model. Because the power demand on average increases daily at the same time that it oscillates around the mean value, the electricity price will vary and, as will the revenues of the utilities.

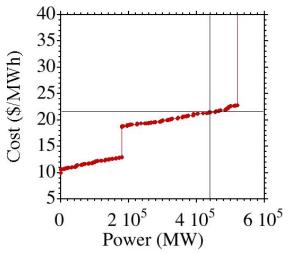


Fig.1 Typical supply function

In the present model, we consider three types of generators. They are a 500 MW coal-fired generator, a 500 MW combined-cycle gas turbine, and a 2000 MW nuclear reactor. For each, we have the cost of the initial capital investment and expected cost of electricity taken from [5, 6]

### 3. The economics model for generation

We allow up to 12 separate utilities to be associated with the  $N_{\rm g}$  generators. The generators associated with a given utility are initially randomly chosen. Following that, they are kept with the same utility throughout the calculations.

There are several parameters associated with each utility. Each utility has a value for its Minimal Acceptable Rate of Return (MARR) to determine its investment strategy, an initial amount capital for starting operations, and a day of the year on which it plans the construction of new generators. The MARR for a utility is determined initially by the formula

$$MARR[i] = \langle MARR \rangle + \Delta MARR(rand - 1/2)$$
 (1)

Here, <MARR> is the average MARR,  $\Delta$ MARR/2 is the range of the MARR among the utilities considered, and rand is a random number between 0 and 1. The initial capital for each utility is similarly assigned. Finally the planning day is randomly assigned to be between 1 and 365.

Every year on its assigned day, each utility analyzes their potential investments. To carry out the investment it must not have any generator under construction and it is discouraged from investing if many other utilities are already building generators. If these two criteria are met, the next consideration is the amount of cash on hand, which guides them toward

one of the three generator types. They can add the new generators to the ones they already have. They are also allowed to borrow money to invest, but in this case, the can add only a single generator.

Each utility uses an estimate of the future price of the electricity, and checks to see if it is profitable to put another generator on line with the electricity cost associated with it. If they find a good option, they analyze the potential of the investment calculating the internal rate of return (IRR) and comparing with their MARR. If the IRR is equal or larger than the MARR, they go ahead and build a new the new generators. After the number of days given in Table I for the type of generation chosen, the generator begins operation in the network.

Table I. Grid utilization for different values of the enhancement factor

F =	2.2	1.8	1.4	1.3	1.2	1.1	1.0
Averaged line loading	0.60	0.596	0.63	0.62	0.56	0.60	0.596
Average line flow limit per MW served	0.08	0.07	0.05	0.03	0.006	0.006	0.004

The estimate of the future price is a critical factor in determining investment; the approach used to make this estimate can change the dynamics. At present, we keep a record of the averaged monthly clearing price, p<sub>i</sub>, for the most recent 12 months. We have used three different methods for estimating the future price:

- 1. A linear fit is made to the data and then extrapolated 12 months ahead, this is  $Fp_1$ .
  - 2. The averaged value of the 12 months is  $Fp_2$ .
- 3. A weighted average is computed in the following way

$$Fp_3 = \frac{1}{66} \sum_{i=1}^{12} i p_i \qquad (2)$$

The results presented here are obtained by averaging  $Fp_1$  and  $Fp_3$ .

To stimulate the investment on new generators, we introduce another mechanism to the model. A critical margin, CM, is defined as a fraction of the total power in the system. When the capacity margin is below this critical margin the clearing price is multiplied by an enhancement factor F. This stimulates investments and accounts for the "missing money" for investment in generation capacity [7]. It can be interpreted as an intervention of the regulatory system or as a change of rules in the electricity market.

# 4. A reference case for the dynamic evolution of the system

Before discussing the dynamics of the economic model for generation coupled to OPA we describe the result of a reference case. In the following calculations we have used the WECC 179 network. We assume that there are 12 utilities associated with the 49 generator nodes.

The basic OPA parameters for all the calculations presented here are: 1) the daily rate of growth in demand,  $\lambda = 1.00005$ , which corresponds to an averaged annual growth of about 2%, consistent with the US data for the last two decades; 2) the rate of upgrade of overloaded lines  $\mu = 1.12$ ; after a blackout, overloaded lines are upgraded at this rate; this value has been adopted to match the averaged frequency of the blackouts in the Western interconnect; 3) the daily load fluctuation parameter,  $\gamma = 1.3$ , the network is divided in four regions and the power demand in those regions varies around the averaged demand by a fraction smaller or equal to  $\gamma - 1$ ; 4) probability of a random line outage  $p_0 = 0.00001$ , and 5) the probability that an overloaded line will outage during a cascading process is  $p_1 = 0.15$ . More details on the role of these parameters in the OPA model can be found in Refs. [1, 2].

As reference values for the parameters of the economic model, we have used:  $\langle MARR \rangle = 0.10$ , CM = 0.3, and F = 1.8. These are not necessarily the optimal values. All calculations were done for a period of 120,000 days.

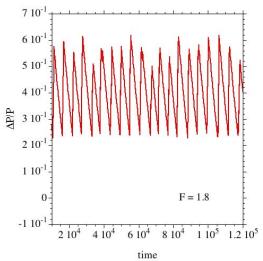


Fig.2 Capacity margin as a function of time for the reference case.

For the base case, the capacity margin is maintained well above zero. The minimum value of the capacity margin over the whole time period is 0.228; the critical margin for the economic incentives needed to maintain this capacity margin is 0.3.

In Fig. 2, we have plotted the evolution of the capacity margin as a function of time. There is a basic oscillation with an averaged period of about 15.5 years.

Note that the typical time scale for this model is  $1/(\lambda-1)$ . If the system has a maximum capacity  $M_1$  at a given time and if the system evolves only as a result of the increase in demand, the time it takes to get at a minimum capacity  $M_2$  is:

$$\tau = \frac{1}{\lambda - 1} \ln \left( \frac{1 + M_1}{1 + M_2} \right) \tag{3}$$

For the particular case that  $M_1 = 0.6$  and  $M_2 = 0.2$ , we get  $\tau = 15.76$  years, which is approximately the value given above. Therefore, if the construction times of the different plants are short compared with this value, construction does not greatly affect the time scale of the oscillations.

This reference case corresponds to a fairly competitive market in which 8 of the 12 utilities are competing at similar level. This can be seen in Fig. 3, where we have plotted the fraction of the total extant capital that each utility holds as a function of time. There are fairly large oscillations but, in the end, each of the 8 surviving utilities held a reasonable fraction of this capital.

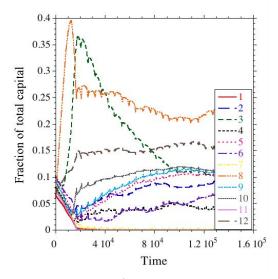


Fig.3 Fraction Extant Capital for the 12 utilities as a function of time for the reference case.

The frequency of blackouts involving outage and/or overloaded lines is 0.015 and the frequency of all blackouts is 0.023. These relatively low values of the frequencies are due to the fact that the probability of the random failures  $p_0$  is rather low.

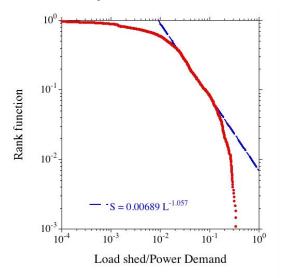


Fig.4 Rank function of blackouts.

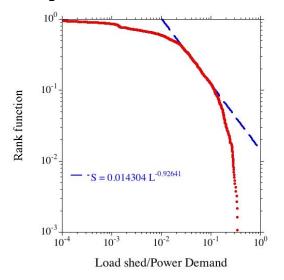


Fig.5 Rank function of blackouts with outages

The rank function of the load shed normalized to the total power has a well-defined power tail with an exponent close to -1, as can be seen in Fig. 4. This value for the exponent is consistent with the one obtained in the analysis of the data from the Western interconnect. If we select the blackout events that involve an outage and/or overload lines, the rank function for the normalized load shed has a similar power law to the one we gat if we take all events. However, the region of power tail seems to be

somewhat narrower in the second case, as shown in Fig. 5.

The events corresponding to large values of load shed are associated with the minima of the capacity margin. In looking at the time sequence of the blackouts, they appear to be bursty and correlated with these minima. This correlation can be seen in Fig. 6. However, as we can see by comparing Figs. 3 and 4, these large events extend the power tail, but they are not necessarily the reason for such power tail.

Because the daily load fluctuation parameter  $\gamma$  is 1.3, the loads fluctuate up to a 30%. Therefore, it makes sense that when the capacity margin goes below 30% blackouts with no outages are often triggered.

We use this case as a reference case in what follows, and we explore in the following sections the dependence of the system performance and reliability in the values of the main parameters controlling the economics model.

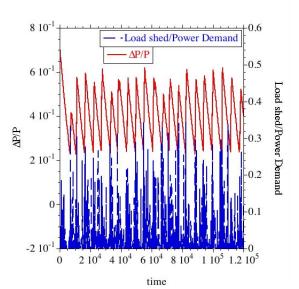


Fig.6 Margin and normalized load shed as function of time showing the correlation between the two.

# **5.** Effect of varying the price enhancement factor

The price enhancement factor F that acts when the capacity margin is below the critical margin increases the clearing price by a factor F, increasing the benefits to the utilities and providing them with some of the so-called "missing money" for investment in generation. If this factor is close to one, the amount of extra cash available to the utilities is small and not surprisingly, it

is not possible to keep the capacity margin above zero. In Fig. 7, we show the sensitivity of the averaged capacity margin to changes in this factor.

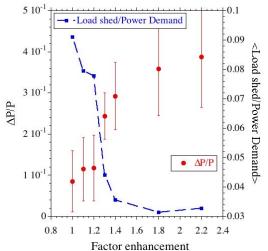


Fig.7 Capacity margin and normalized load shed plotted vs. price enhancement factor

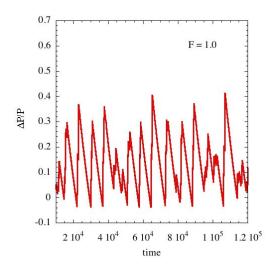


Fig.8 Capacity margin as a function of time

The error bars represent the standard deviation in the fluctuations in the capacity margin; they give a measure of the averaged size of its oscillations. In the same figure, we also show the averaged load shed normalized to the power demand per blackout.

In examining Fig. 7, we can see that when the factor is above 1.4, there is a relatively stable regime with a capacity margin always above 20%. When the factor is bellow 1.4 there are problems; the capacity margin is not always maintained above zero. When the factor is below 1.2 the system collapses and the mechanism to stimulate investments no longer has any

positive effect. When F is 1, there is no enhancement of the clearing price and the capacity margin drops below 0.4 and becomes negative, as can be seen in Fig. 8.

In the transition region, when 1.5 > F > 1, there is not only a change in the mean value of the parameters and in the properties of the blackouts, but also a change in the economic dynamics. This is reflected in the change of the time evolution of the capacity margin. Although the capacity margin is sometimes pushed above 0.2, as in Fig. 2, there are periods of time that it becomes negative, as shown in Fig. 8.

When the price enhancement factor decreases, not only the average size of blackouts increases but also the frequency of blackouts. The reliability of the system deteriorates seriously. In Fig. 9, we have plotted the frequency of all blackouts and of those blackouts that have outage or overloaded lines.

As the enhancement factor increases, there is a decrease of the blackouts without outage or overloaded lines (brown outs) but also of the blackouts with outage lines. When the system collapses because of the lack of investments, the frequency of the brown outs gets close to 1; brown-outs occur nearly every day.

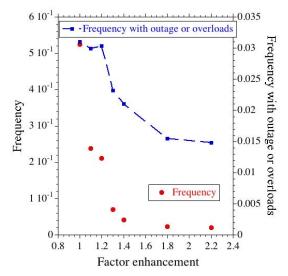


Fig.9 Frequency of blackouts (total and with outage) plotted vs. price enhancement factor

The distribution function of the blackout size, measured here by the load shed normalized to the power demand, also changes as F changes. For the base case, F = 1.8, we have seen that the Rank function had a clear power law region with exponent -1. As F decreases, the Rank function is broader and at a certain point the power law region disappears. This is shown in Fig. 10. If we examine the same Rank function but only for the blackouts with outage or overloaded lines,

we see that the power law region is maintained up to F = 1.3, with the same exponent. Only below 1.3 does the Rank function change to exponential. Of course, one of the reasons for the change to exponential tail is that most of the blackouts are of the order of the system size and a power tail behavior is no longer possible.

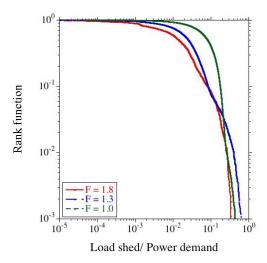


Fig.10 Rank function of blackouts for 3 values of F

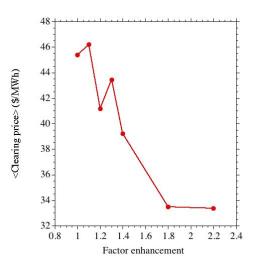


Fig.11 Clearing price as a function of enhancement factor

As the clearing price enhancement factor decreases, there is a serious deterioration in system reliability as shown by an increase in the frequency and size of blackouts. This decrease in reliability is accompanied by an apparent paradox: an increase of the cost of

power to the consumer. Reducing the enhancement factor does not save money, because the system spends more time bellow the critical margin. If we plot the averaged clearing price as a function of the enhancement factor, we can see this increase in cost. This plot is shown in Fig. 11.

In Ref. [8], we introduced a measure of utilization of the system, which is the average line flow limit per MW served. It is defined as,

Averaged line flow limit per MW served at time t = 
$$\frac{1}{N_{lines}} \sum_{j=1}^{N_{lines}} F_j^{Max}(t)$$
 (4)

Here,  $F_j^{Max}(t)$  is the power flow limit of the line j at time t, and  $N_{lines}$  is the number of lines in the network.

In Table I, we show the change in grid utilization as the price enhancement factor changes. The average line load is not very affected by changes in F; however, the utilization of the system deteriorates as F is reduced.

For values of the factor above 1.4, we get similar results to the results obtained for the IEEE 118 network when the n-1 criterion was applied [8]. Below F = 1.4 there is a reduction in the utilization by a factor of 2 or 3.

## 6. Effect of varying the critical margin and the averaged MARR

Another important parameter in this model is the critical margin. This is the value of the capacity margin below which the clearing price is enhanced by the factor F. Changing the critical margin has the expected impact on the system. It changes the capacity margin and the size of the blackouts. Both change in an expected way as shown in Fig. 12.

In Fig. 12, the error bars in the capacity margin represent the standard deviation; they reflect the amplitude of the oscillation of the capacity margin. The changes are less dramatic than they were when case the enhancement factor was changed, but the impact is similar.

The effect on the frequency of the blackouts is also similar. As the critical margin decreases, there is a significant increase in the brown-outs; the frequency of the blackouts with line outages also increases. This is shown in Fig. 13

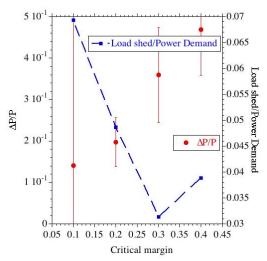


Fig.12 Capacity margin and normalized load shed plotted vs. critical margin

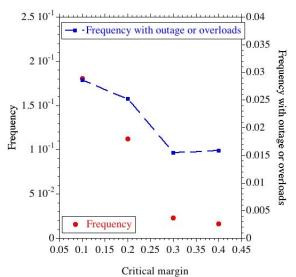


Fig.13 Frequency of blackouts (total and with outage) plotted vs. critical margin

Increasing the averaged MARR, <MARR>, of the utilities very quickly leads to reliability problems. As <MARR> increases, it becomes more difficult to get enough cash for investments in generation, because the utilities do not want to take these risks. Therefore, there is no way of maintaining a reasonable capacity margin. The system then has large and frequent blackouts.

In Figs. 14 and 15, we provide plots <MARR> of the same data as we did for the other parameters. The patterns of time evolution of the capacity margin are reminiscent of the ones in the transition region when

the enhancement factor was varied. They show the capacity deepening below zero and a burst of blackouts emerging. This is typical for this model when not enough investments in generation are made.

Also in this case, the decrease in reliability of the system is concomitant with a large increase in the cost of electricity. The clearing price goes from \$33.48 per MWh in the base case, to a \$59 per MWh in the case of the highest <MARR>.

### 7. Transients in the dynamic evolution

Though the economic model discussed here is rather simple, it has a complex dynamics. From the previous analysis we can see that there are two stable steady state solutions. They are exemplified in Figs. 2 and 8. One of these states corresponds to the solution of the reference case with the capacity margin kept above 0.2. The other is the state that corresponds to F = 1.0 with the capacity margin going below zero.

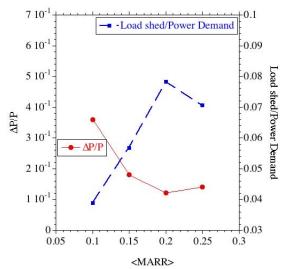


Fig.14 Capacity margin and normalized load shed plotted vs. avg MARR

In addition, there are solutions that seem to jump between these two types. In many cases the solution keeps the capacity margin above a given value, but for a period of time, the capacity goes below zero. This type of behavior is generally found in the transition region of the enhancement factor and in the cases where MARR takes on higher values, but they can also be triggered by changes in the economic conditions.

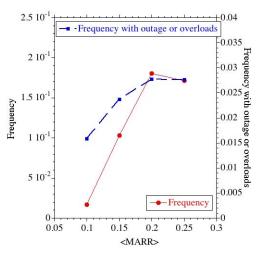


Fig.15 Frequency of blackouts (total and with outage) plotted vs. price enhancement factor

For instance, we can start from the reference case described in Section II and run it for 30000 days. At this point, the enhancement factor is changed from 1.8 to 1.4. Therefore, the extra money the utilities get is reduced by one half. Here we call the reference case 1 and the one where we change F at t = 30000 case 2. A plot comparing these two cases in terms of the time evolution of the capacity margin is given in Fig. 16.

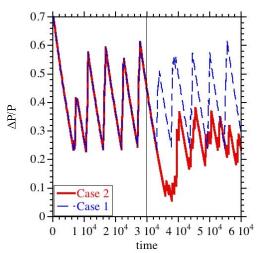


Fig.16 Capacity margin as a function of time, reference case and transient case.

In Fig. 16, the effect of changing F is clear; the capacity margin drops below 10%. After about 5000 days it recovers and oscillates at a lower level corresponding to the reduced value of F. This transient is associated with a sudden burst of blackouts. This is shown in Fig. 17, where we have plotted the

normalized load shed as a function of time. In this figure and at t = 30000, the load shed increases in size and frequency.

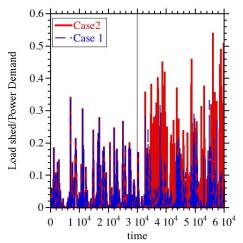


Fig.17 Normalized load shed as a function of time, reference case and transient case.

There is a drop in the reliability of the system at the point where the factor F is changed. The system changes some more some time later, when the price enhancement factor only goes into operation when the capacity margin falls below 0.3.

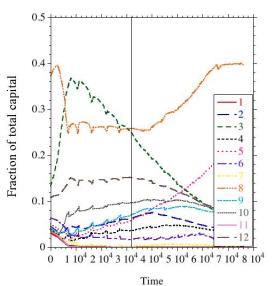


Fig.18 Market fraction for the 12 utilities as function of time for the transient case.

The basic parameters describing the reliability of the system all change, as shown in Table II. There is also a change in the dynamics of the utilities after the change in F as shown in Fig. 18.

Table II. Change of the reliability parameters and grid utilization during a transient

	1st 30000 days	30000 days post change	
Frequency blackouts	0.022	0.119	
<load power="" shed=""></load>	0.027	0.061	
Average line flow limit per MW served	0.061	0.0152	

#### 8. Conclusions

The interaction between a dynamic model of the power transmission system (OPA) and a simple economic model of power generation development is found to lead to complex dynamics both in the economics (prices, market share etc) and in the transmission system characteristics (blackouts, reliability etc). The most effective control parameter, the price enhancement factor, moves the system from a critical state, for larger values of F, to a continuously failing system for small values of F. The other control parameters, the critical margin and the Minimal Acceptable Rate of Return, have similar but less dramatic effects. They can also move the system into different states with vastly differing properties. These states are characterized by power law tails in the failure sizes in one limit and exponential tails with extremely high frequency of failures in the other limit. Because the price enhancement factor and the critical margin can be set by regulators, they could therefore be

directly influenced by reliability considerations. The next step in this investigation is to add regulatory feedback on these parameters to investigate the impact on the overall system reliability.

### Acknowledgements

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